MEĐUNARODNI NAUČNI SKUP INTERNATIONAL SCIENTIFIC CONFERENCE

"DANI ARČIBALDA RAJSA" "ARCHIBALD REISS DAYS"

Beograd, 1-2. mart 2013. Belgrade, 1-2 March 2013

TEMATSKI ZBORNIK RADOVA MEĐUNARODNOG ZNAČAJA

THEMATIC CONFERENCE PROCEEDINGS OF INTERNATIONAL SIGNIFICANCE

TOM I VOLUME I

KRIMINALISTIČKO-POLICIJSKA AKADEMIJA Beograd, 2013 ACADEMY OF CRIMINALISTIC AND POLICE STUDIES Belgrade, 2013

Publisher ACADEMY OF CRIMINALISTIC AND POLICE STUDIES Belgrade, 196 Cara Dušana Street (Zemun)

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Computer Design MILOŠ IVOVIĆ/MILAN SREĆKOVIĆ

Impression 200 copies

Print SLUŽBENI GLASNIK, Belgrade

THE CONFERENCE AND THE PUBLISHING OF PROCEEDINGS WERE SUPPORTED BY THE MINISTRY OF EDUCATION AND SCIENCE OF THE REPUBLIC OF SERBIA

© 2013 Academy of Criminalistic and Police Studies, Belgrade

ISBN 978-86-7020-190-3 ISBN 978-86-7020-260-3

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Tehničko uređenje MILOŠ IVOVIĆ/MILAN ŚREĆKOVIĆ

> Tiraž 200 primeraka

Štampa JP "SLUŽBENI GLASNIK", Beograd

ODRŽAVANJE SKUPA I ŠTAMPANJE OVOG ZBORNIKA PODRŽALO JE MINISTARSTVO PROSVETÉ I NAUKE REPUBLIKE SRBIJE

© 2013 Kriminalističko-policijska akademija, Beograd

ISBN 978-86-7020-190-3 ISBN 978-86-7020-260-3

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INTRODUCTORY PAPERS

APPLICATION OF GAME THEORY IN THE MODELING FUNCTION OF POLICE NEGOTIATION¹

Associate Professor Dane Subošić, PhD Associate Professor Obrad Stevanović, PhD Assistant Professor Stevo Jaćimovski, PhD Academy of Criminalistic and Police Studies, Belgrade

Abstract: Game theory is a mathematical theory of conflict situations between rational decision-makers, whose decisions affect each other. In addition, the conflict situations are characterized by two (or more) opposing sides, with antagonistic goals (mutually opposed and irreconcilable); where the result of each action of participant depends on what action the opponent will choose. Due to the antagonism of parties objectives involved in the negotiations, negotiation is particularly suitable for modeling activities by means of the theory of games. Of special interest to the police organization is the police negotiation organization. In this context, this paper focuses on the modeling of police negotiation through the game theory.

Keywords: game theory, police negotiation, mathematical modeling.

INTRODUCTION

Game theory is a branch of mathematics used to analyze competitive situations ("games") which outcomes depend not only on one's own choices, and perhaps chance, but also on the choices made by other parties ("players").² In addition, game theory concerns the behaviour of decision makers whose decisions affect each other.³ In the field of economy, these situations are called competition. Game theory was created in 1928 when von Neumann proved the theorem on minmax. Later it began to be applied in many other branches of science and life - almost every time you need to develop a strategy in the conflict of interest situations. In the book "Theory of games and Economic Behavior" in 1944, this theory was developed by the economist Oskar Morgenstern and the famous mathematician John von Neumann⁴. They noted that in the economic sciences there are no mathematical models good enough to describe situations in which market participants are facing mutual conflict interests.

Therefore, Neumann and Morgenstern described these situations by abstracted games, conceived as a set of rules and conventions that players must follow. At each stage of the game, players pull certain moves by the set of (un) limited decisions making the right choices and choose that decision that seems best to them on the basis of available information. Fundamental contributions to the theory of games were made by John Nash⁵.

Game theory is the basis for many theories of negotiation used by police and diplomacy. Strategies, developed in the game theory by appropriate mathematical tool, offer players the instruction set for any situation that may arise during a game, and one of the key aspects for making right

This paper is the result of the research on the following projects: "Development of Institutional Capacities, Standards and Procedures for Fighting Organized Crime and Terrorism in Climate of International Integrations", which is financed by the Ministry of Education and Science of the Republic of Serbia (No 179045), and carried out by the Academy of Criminalistic and Police Studies in Belgrade (2011-2014). The leader of the Project is Associate Professor Saša Mijalković, PhD, "Structure and functioning of the police organization - transition, condition and perspectives", which is financed by the Academy of Criminalistic and Police Studies; "Violence in Serbia – Causes, Forms, Consequences and Social Response", which is financed by the Academy of Criminalistic and Police Studies; "Innovation of forensic methods and their application" (No

nnanced by the Academy of Criminalistic and Police Studies; Innovation of robresic methods and their application (No TR34019), which is financed by the Ministry of Education and Science of the Republic of Serbia.

2 See also: Brams J. S., *Game Theory, International Encyclopedia of the Social Sciences, 2nd ed., forthcoming,* New York University, New York, 2005, 23 February 2013 https://www.nyu.edu/gsas/dept/politics/faculty/brams/GameTheory.pdf. Put a Aumann, R.J. "game theory." The New Palgrave Dictionary of Economics. Second Edition. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan, 2008. The New Palgrave Dictionary of Economics Online. Palgrave Macmillan, 23 February 2013 https://www.dictionaryofeconomics.com/article?id=pde2008_G00007

Neumann J. von, Morgenstern O., *Theory of Games and Economic Behavior*, Princeton University Press, 1953 Nash J. Non-cooperative games, *Annals of Mathematics* 54 (1951), pp. 286-295

decision is to consider the possible moves of rival players. In 1994 "for a pioneering analysis of equilibrium in the non-cooperative games theory" John Nash, John Harnasnhi and Reinhard Zelten received the Nobel Prize in economics. In 2005 for "increasing *our understanding of conflict and cooperation through game-theory analysis*" Robert Auman and Thomas Schelling received the Nobel Prize in economics.

The paper will discuss the theory of games in the function of modeling of police negotiation for resolution of conflicts situations. The police have a role in resolving conflict situations in the field of security. In doing so, it has a duty to protect also the security of holders of threat. Minimization of harmful effects to the life and health of all participants in the conflict requires the application of police methods in the resolving non-compulsory conflict situations. One of these methods is negotiation.

Negotiation is a process of mutual persuasion of opposing sides in the communication in terms of antagonism of their goals. Police (security) negotiation is usually carried out in the cases of kidnapping, hostage situations, severe forms of extortion, riots in prisons, occupation of buildings, street demonstrations, threats of suicide and homicide (murders), threats to the police or third parties by weapons or explosive devices in the preparation and implementation of police measures, resistance to police measures etc., when it is possible to influence the behavior of perpetrators of conflict situations, with a view to their withdrawal from illegal conducting⁶.

Due to the complexity of circumstances in which police negotiation is implemented, it is necessary for its modeling, to predict its effects to the greatest possible certainty. Maximization of police negotiation effects is achieved by optimal strategies through which it realizes. Mathematical (exact) method to do this is the application of game theory. This paper questions the right application of game theory in the modeling function of police negotiation.

BASIC TERMS OF THE GAME THEORY

Basic terms of the game theory are:

- 1) Game,
- 2) game with zero-sum
- 3) move
- 4) strategy
- 5) optimal strategy
- 6) pure strategy
- 7) mixed strategy
- 8) low value of matrix game
- 9) high value of matrix game
- 10) Saddle point of matrix game
- 11) value of game
- 12) goal of game theory

Game is a model of conflict situation.

Game with a zero-sum is that game in which the payoff of a participant is identical to the payoff loss of another participant.

Move is a selection of one of the possible alternatives available to participants in the game.

Strategy is a set of rules that uniquely determine the choice of gait of each participant in the game.

Optimal strategy is a kind of strategy that during multiple repeated games provides the participant to achieve the maximum possible medium payoff, i.e., minimum possible medium loss. In the game theory there is already mentioned principle (criterion) of the minimax. It is expressed by the view that a player in the matrix game (conflict situation simulation) chooses his behavior in a way that maximizes his payoff with, for him, the most adverse action of opponent. By this principle the

⁶ Subošić D., Organizacija i poslovi policije, Kriminalističko-policijska akademija, Beograd, 2010.

choice of the most cautious strategy for each player is conditioned. At the same time, the minimax is also the basic principle of game theory. Strategies chosen trough this principle are called minimax strategies⁷. Pure strategies are at least one strategy that both players have at their disposal, which are predicted to be better than all the strategies of opponents.

Mixed strategy is a complex strategy, which consists in applying more pure strategies in a certain respect. This type of strategy can be reached via the selection probability of one pure strategy (p_1) and selection probability of other pure strategy $(p_2=1-p_1)$. At the same time, the value of p_1 is in the range from 0 to 1.

The low value of matrix game is the maximum payoff between minimum payoffs (α).

The high value of matrix game is the minimal loss between maximal effects (β) .

The saddle point of matrix game is that point which is expressed by the maximum in its column and by the minimum in its row⁸. It exists in the case when the low and high value of matrix game are of identical values ($\alpha = \beta$). If α is not equal to β , there are no saddle. The difference between α and β is "space" in which the participants in the matrix game should demonstrate own abilities, by choice of optimal - mixed strategy.

The value of game (ν) is the value between the maximum payoff between minimum payoffs (α) and minimal loss between maximum effects (β) . Mathematically expressed, the value of game is the interval: a £ n £ b.

The goal of game theory is the determination of optimal strategy for each participant.

CLASSIFICATION OF MATRIX GAMES

Matrix game is a game that can be realized by the following rules:

- 1) The game involves two players
- 2) Each player has a finite set of available strategies
- 3) The game consists in the fact that each player having no information about the intentions of opponent makes a move (chooses one of the strategies). As a result of the chosen strategy payoff or loss in the game arises.
- 4) Both payoff and loss in the game are expressed as a number

The strategy of player I will be seen as the row of some matrix, and the strategy of player II as the column of some matrix. The situations in the game are presented by boxes at the intersection of rows and columns. Filled boxes – situations – by real numbers that represent the player's I payoff, we give a task in the game. Resulting matrix is the winning matrix of game or game matrix. Taking into account the antagonism of matrix game, the payoff of player II in any situation means the loss of player I and differs only by sign. No additional explanation on the function of winning player II is required. The matrix that has m rows and n columns is called $(m \times n)$ matrix, and game $(m \times n)$.

There are simple and mixed matrix games. They differ in how the simple games have and mixed have not so-called "saddle point". In addition, the simple matrix games correspond to the situations of certainty, while the mixed situations correspond to the situation of uncertainty. In the connection to it the next image is given.

⁷ Petrić, J., Šarenac, L., Kojić, Z.: Operaciona istraživanja, zbirka rešenih zadataka, knjiga 2, Naučna knjiga, Beograd, 1988.

⁸ Milovanović, M.: Odlučivanje u borbenim dejstvima, studija, Vojna akademija, Beograd, 2004.

⁹ Ibid

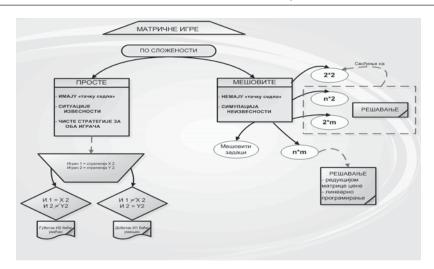


Figure 1. Classification of matrix games¹⁰

MODELING OF THE POLICE NEGOTIATION THROUGH THE MATRIX GAMES WITH SADDLE

For the games with a saddle is characteristic that they have clear strategies for both players, which by row and column correspond to the saddle point. The example of the police negotiation (see chart below) means that the police (player no. 1) has at its disposal two strategies (x_1, x_2) , while the opposing party (player no. 2) also has at its disposal two strategies (y1, y2). For example, both strategies for both parties are indulgent and unyielding negotiation.

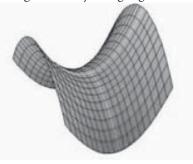


Figure 2. Saddle point

Table 1. Player strategy

Players	Alternatives			
I	x_1	x_2	x_3	
II	y_1	y_2	y_3	

¹⁰ Subošić, D, Daničić, M.: Bezbjednosni menadžment, organizacija i odlučivanje, Fakultet za Bezbjednost i zaštitu, Banja Luka, 2012, p. 243.

In general case it means that he has at least two strategies

Let's take a concrete example. Game payoff for the player I, which depends on the choice of possible strategies, is given by the following set of data,

$$v(x_1, y_1) = 4$$
, $v(x_1, y_2) = -1$, $v(x_1, y_3) = -4$, $v(x_2, y_1) = 3$, $v(x_2, y_2) = 2$ $v(x_2, y_3) = 3$, $v(x_3, y_1) = -2$, $v(x_3, y_2) = 0$ $v(x_3, y_3) = 8$.

Find a solution to a game, i.e. determine:

- a) optimal strategic pair (x, y),
- b) find the value of matrix game

Resolution: The game defined above can be reduced to the matrix form, as shown in Table 2, where rows correspond to the possible strategies of player I and column to the possible strategies of player II.

			•		
		Player strategy II			Minimum by rows
		y_1	y_2	y_3	Transmission by Torro
Player strategy I	$x_{_{1}}$	4	-1	-4	-4
	x_2	3	2	3	2
	$x_{_3}$	-2	0	8	-2
Maximum by column		4	2	8	

Table 2. Game with saddle point

By analysis of the matrix game price, the player I determines that if he chooses strategy x_1 at least he will get is -4, for strategy x_2 is 2 and if he chooses strategy x_3 minimum payoff is -2. The player I will try to choose such a strategy which corresponds to the maximum among specified minimum payoffs. In our case, it is strategy x_2 .

The payoff value, which corresponds to strategy x_2 , is called the low value of game and is marked with α . Therefore, we have that

$$\alpha = max_i min_i (a_{ii}) = 2$$

By analysis of the matrix game price, the player II determines that if he chooses strategy y_1 the maximum what he can lose is 4, for the strategy y_2 is 2, and if he chooses strategy y_3 the maximum that he can lose is 8. The player II will try to choose such a strategy corresponding to the minimum of specified maximum losses in each column. In our case it is strategy y_2 . Thus, the value obtained is called the high value of game and is marked with β . Therefore, we have that

$$\beta = min_i max_i (a_{ii}) = 2$$

If the high value of game is equal to the low value of game for such matrix game is said to have a saddle, and solution to the game is in the domain of pure strategies. In other words, if both players find at least one strategy that is the best according to prediction in relation to all the strategies of his opponent it is said that the game has as solution the pure strategies as optimal. This is possible only if the matrix game has a saddle. In this case, the value of game is

$$v = \alpha = \beta$$
.

In our case, the matrix game has the saddle and optimal strategies are in the domain of pure strategies, namely:

I player - strategy
$$x_2$$
,
II player - strategy y_2 ,

and the low value of game is equal to the high value of game, i.e.

$$v = \alpha = \beta = 2$$
.

The element $a_{22}=2$ in the matrix a_{ij} is called the saddle of matrix game. If the player I applies any other strategy, not the strategy x_2 , and player II remains at the optimal strategy, the payoff of the player I will be reduced. Also, if the player II tries any other strategy and not y_2 , and the player I maintains its optimal strategy; the loss of player II will be increased 12.

¹² Petrić, J., Šarenac, L., Kojić, Z.: Operaciona istraživanja, zbirka rešenih zadataka, knjiga 2, Naučna knjiga, Beograd, 1988.

POLICE NEGOTIATION MODELING THROUGH MIXED MATRIX GAME

Mixed matrix games are divided into: matrix game (2×2) , matrix game $(n \times 2)$, matrix game $(2 \times m)$ in $(n \times m)$. Solving the matrix games $(n \times 2)$ and $(2 \times m)$ is conducted by reducing them to the matrix game (2×2) . Solving the matrix games $n \times m$ can be done by reducing the price matrix and linear programming. Finally, in addition to the above, there are also mixed tasks.

Mixed matrix games (2×2) are the matrix games types characterized by existence of two pure strategies of each participant (police negotiator and police opponent), and there is no" saddle", which is why their solution lies in the field of mixed strategies. Solving the matrix game is shown in the following example.

While the party I (police) calls a dangerous criminal whose arrest is in progress, to surrender, the party II (dangerous criminal) refuses to surrender. In order to minimize harmful effects to the life or health in this conflicting situation, there comes to negotiation. The task is: find the low and high value of matrix game (α and β), determine the optimal strategies of participants and the value of game (P, Q, v). The probabilities for all combinations of opposing party's strategies are given in the following table.

Table 3: The probabilities for all combinations of opposing parties' strategies

	y_1	y_2	Minimum by row
x_{i}	0,4	0,2	$\alpha = 0.2$
x_2	0,2	0,6	0,2
Maximum by column	$\beta = 0.4$	0,6	

The high and low value of matrix game has no identical values ($\alpha \neq \beta$), so that the matrix game does not have: "saddle" It also means that the matrix game has an optimal strategy in the mixed strategies domains. Therefore, it is necessary to calculate the mixed strategy vectors P and Q, as it in the requested task.

The vector of mixed-strategy negotiation (police) party I is $P = \begin{bmatrix} p_1 & p_2 \end{bmatrix}$ where

respectively

On the other hand, the vector of mixed strategies by police opponent II is $Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$, where $q_1 + q_2 = 1$,

respectively

$$q_1, q_2 > 0.$$

This means that the party I (police negotiator) will choose strategy x, with probability, p, and this probability is higher than zero and so on. The matrix game value, for general case, is defined by the following equation:

$$v(P,Q) = \sum_{i=1}^{n} \sum_{j=1}^{m} P \cdot A \cdot Q = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} p_{ij} q_{ij}$$

The table $A = [a_{ij}]$ is called a payoff matrix and with it the game is entirely determined. In general, a zero sum game in which the first player has m pure strategies and the second player with n pure strategies is given by $m \times n$ matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

The rows correspond to the strategies of first player, the column to second player, and the number a_{ij} determines payoff to the first player if he chooses i-th and his opponent j-th strategy. Positive number means that player gets the appropriate amount, and negative that he loses. **Von Neumann Theorem**: For every zero - sum game for two players, there is a pair of strategies P_0 , and number v with properties¹³:

- 1) If the first player uses the strategy P_0 his expected profit (mathematical expectation) is at least v, ie. $E(P_0,Q) \ge v$ for each Q;
- 2) If the second player uses the strategy Q_0 expected profit (mathematical expectation) of the first player is not greater than ν , ie. $E(P,Q_0) \le \nu$ for each P;

Due to this theorem the terms of optimal strategy and the values of game make sense for all the games, not only for strictly defined. Rational players will choose strategies P_0 , Q_0 and then the expected payoff of the first player will be equal to the value of game v. If the first player chooses a strategy that is not optimal, his expected payoff can be greater than v for some strategies of second player, but there is the strategy Q for which the expected payoff is less than v. Analogously, if the second player chooses a strategy that is not optimal, the first player has strategy P in which the expected payoff is greater than v.

Von Neumann's theorem can be proved by reduction to the linear programming problem. Linear programming deals with the minimization and maximization of linear functions of several variables which are subject to the conditions set by linear inequalities. There are more efficient algorithms for solving such problems, among them the simplex method is the most popular. It is shown that for the game with payoff matrix $A = [a_{ij}]$ finding the optimal strategy of the first player R $P_0 = [n_1 \ n_2 \ ... \ n_m]$ is equivalent to the linear programming problem¹⁴:

$$v \to \max v \to \max v \to a_{1j} p_1 + ... + a_{mj} p_m, j = 1,..., n$$

$$p_1 + ... + p_m = 1$$

$$p_i \ge 0, i = 1,..., m$$

Finding the optimal strategy of second player Q_0 is equivalent to the following linear programming problem.

$$u \rightarrow \min u \ge a_{i1} q_1 + ... + a_{in} q_{in}, i = 1,..., m q_1 + ... + q_n = 1 q_j \ge 0, j = 1,..., n$$

According to one of the fundamental theorem of linear programming, duality theorem, the maximum from the first problem is equal to the minimum of second problem, v = u. It is the value of game, and amount of variables $p_1, ..., p_m$ and $q_1, ..., q_n$ (for which the maximum and minimum are achieved), are the components of optimal strategy for the first and second player.

The specific problem given in Table 3 can be solved by means of mathematical package *Mathematica* 8.

```
\label{eq:maximize} \begin{split} &\text{Maximize}[\\ & \{v,\,v \le 0.4\,p_1+0.2\,p_2,\,v \le 0.2\,p_1+0.6\,p_2,\,\,p_1+p_2=1,\,p_1 \ge 0,\,p_2 \ge 0\},\,\{v,\,p_1,\,p_2\}] \\ & \{0.3333333,\,\{v \to 0.3333333,\,p_1 \to 0.666667,\,p_2 \to 0.3333333\}\} \end{split} \begin{aligned} &\text{Minimize}[\\ & \{u,\,u \ge 0.4\,q_1+0.2\,q_2,\,u \ge 0.2\,q_1+0.6\,q_2,\,\,q_1+q_2=1,\,q_1 \ge 0,\,q_2 \ge 0\},\,\{u,\,q_1,\,q_2\}] \\ & \{0.3333333,\,\{u \to 0.3333333,\,q_1 \to 0.666667,\,q_2 \to 0.3333333\}\} \end{aligned}
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At the same time, the value of matrix game is:

$$n = 1/3 (0,33)$$
.

¹³ Petrosyan L.A., Zenkevich N.A., Semyna E.A., Game theory, Higher School, Moscow, 1998.

¹⁴ Danilov V.I., Lectures on game theory, New Economic School, Moscow, 2002.

Checking this option, we can make the following comparison:

a£n£b.

i.e. replacing the given values by calculated, we get:

0,2 £ 0,33 £ 0,4.15

CONCLUSION

The study tested and confirmed the applicability of game theory in the function of police negotiation modeling. In doing so, the police negotiation is modeled by the simple and mixed matrix games with zero-sum. The importance of obtained knowledge in this work is evident in yet untapped possibilities for the optimization of police negotiation in the terms of choice of negotiators' optimal strategy.

The application of game theory during police negotiation allows the maximization of police negotiation effects from one and minimization of counteracts to the police, on the other hand. Thus, as the negotiation is very uncertain police activity, which may cause serious consequences in solving the most complex security tasks, it becomes more effective and efficient, primarily due to the optimization and predictability. Therefore, the development of police negotiation may be partly ensured by application of game theory, by development of proper databases on conflict situations and by training of police negotiators for their mathematical modeling.

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¹⁵ Petrić, J., Šarenac, L., Kojić, Z.: *Operaciona istraživanja*, zbirka rešenih zadataka, knjiga 2, Naučna knjiga, Beograd, 1988.